Composing Composers

Joshua Edward Pearson

pearsonj@mit.edu

Abstract. Suppose you believe on independent grounds that Verdi is Italian, Bizet is French, and Satie is French. To your surprise, you then learn that all three composers are compatriots. What should you believe? Some argue you should be *cautious* and become ambivalent as to whether all three composers are French or all three are Italian. Surprisingly, if that's right, Composers *wreaks epistemological anarchy*: a wide variety of epistemological principles turn out to be false. Those resistant to this anarchy instead argue you should be *bold* and conclude that all three composers are French. I endorse the anarchy. But I do so here in a unique way. Existing approaches to Composers side exclusively with either the *cautious* or *bold* reaction, ruling the other out as irrational. As Stalnaker observes, this is undesirable: *both* judgements look reasonable. I outline a new approach to Composers that successively captures this permissive element of the case.

Inspired by a case from Ginsberg (1986) concerning counterfactuals,[1] Stalnaker (1994) outlined the now (in)famous "Composers" case which, if what I call the "cautious" judgements concerning it are correct, *wreaks epistemological anarchy*: various theories of belief revision—e.g. (Alchourrón, Gärdenfors, and Makinson 1985), (Lin and Kelly 2012), (Leitgeb 2014) and (Goodman and Salow forthcoming)—are false, as is the thesis that rational belief supervenes on rational credence. In contrast, the "bold" judgements regarding Composers cause no such trouble. I endorse the anarchy. But I do so here in a unique way. All proposed models of Composers so far side exclusively with either the cautious or bold judgements. As Stalnaker (2019, p. 65) suggests, this is undesirable: *both* judgements seem reasonable, and which is to be preferred depends only on how epistemically bold or cautious the agent in question is. I defend a new approach to Composers that can predict this permissive element,

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[1]Who in turn took inspiration from Quine (1950).

while still maintaining a substantive connection between rational belief and rational credence. §1 considers the cautious judgements; §2 considers the bold judgements; §3 considers the incomparability models of composers outlined in (Lin 2019); §4 outlines my new approach.

1   Cautious Composers

Here's the case:

> **Composers.** You justifiably believe (at time t_0) that Verdi is Italian, Bizet is French and Satie is French. These three beliefs are formed on equally strong, independent bases. At t_1, you learn that Verdie and Bizet are compatriots. At t_2, you learn that *all three* composers are compatriots. (Stalnaker 1994, p.19)

How should your beliefs change throughout Composers? A common reaction is as follows.[2] At t_1 you learn, contrary to what you believed at t_0, that Verdie and Bizet are either both Italian or both French. Since your initial basis for thinking that Verdi is Italian was just as strong as your initial basis for think that Bizet is French, you should suspend judgement on whether they are both Italian or both French. Meanwhile, since your belief Satie is French was formed independently, you should continue to believe that Satie is French. At t_2, you learn that *all three* composers are compatriots. So, Verdi, Bizet and Satie are either all Italian or all French. Can you believe they are all Italian? No. Initially you believed that Verdi is Italian and that Bizet and Satie are French, so to conclude they are all Italian would be to draw the odd conclusion that you were wrong about *both* Bizet and Satie, but not Verdi. At the same time, you are also not in a position to conclude they are all French. While this situation is perhaps more likely than the one in which they are all Italian—it entails only that you were wrong about one composer (Verdi) rather than about two (Bizet and Satie)—it nevertheless seems that the possibility you were right about Verdi but wrong about both Bizet and Satie is one you should leave open. You should therefore suspend judgement at t_2 as to whether they are Italian or all French. Summarising:

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[2]Endorsed in, for example, (Stalnaker 1994) and (Schultheis 2018, Ch. 2), and taken by Lin (2019) and Goodman and Salow (forthcoming) to at least be intuitively plausible.

CAUTIOUS

In Composers, at t_1, you should suspend judgement on whether Verdi and Bizet are both Italian or both French, but you should continue to believe Satie is French. At t_2, you should suspend judgement on whether all three composers are either all Italian or all French.

CAUTIOUS should seem like a perfectly ordinary, innocent, and well, *mundane* reaction to Composers.

Not so. If CAUTIOUS is right, epistemology anarchy ensues. Consider first the central tenet of (Alchourrón, Gärdenfors, and Makinson 1985) and (Leitgeb 2014):

PRESERVATION

If you are justified in believing q and are not justified in believing not-p, then you would still be justified in believing q were you to learn p as total information.

CAUTIOUS constitutes a counterexample to PRESERVATION. At t_1, you are ambivalent as to whether Verdi and Bizet are both Italian or both French, and you believe Satie is French. You therefore leave open the possibility that all three composers are French, and hence that all three are compatriots. So, upon learning at t_2 that all three composers are compatriots, PRESERVATION says you need not give up any of your beliefs, including your belief that Satie is French. CAUTIOUS denies this. Since at t_2 you know all three composers are compatriots, believing Satie is French requires believing all three are French, yet CAUTIOUS says you should be ambivalent as to whether they are all Italian or all French.

As Ginger Schultheis has observed,[3] CAUTIOUS also constitutes a counterexample to the weaker:[4]

ANTICIPATION

If you would not be justified in believing p were you to learn that e as total

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[3]See (Goodman and Salow forthcoming, §4), who credit the observation to Ginger Schultheis via personal communication.

[4]ANTICIPATION is sometimes called "Negation Rationality"; (Kraus, Lehmann, and Magidor 1990) (Freund and Lehmann 1996). Goodman and Salow (forthcoming) discuss a generalisation called 'Π−'.

information, and you would not be justified in believing *p* were you to learn not-*e* as total information, you cannot *now* be justified in believing *p*.

For consider an alternative version of Composers where at t_2, instead of learning that all the composers are compatriots, you learn that Satie is a *different* nationality to both Verdi and Bizet. So either Verdi and Bizet are both Italian and Satie French—meaning you were wrong only about Bizet—or Verdi and Bizet and both French and Satie Italian—meaning you were wrong about both Verdi and Satie but not Bizet. This situation is analogous to the original: although the possibility in which you were wrong about just one composer is presumably more likely than the possibility in which you were wrong about two, you should nevertheless leave this latter possibility open. Hence, at t_2 you should give up your belief that Satie is French. ANTICIPATION therefore fails at t_1: you believe Satie is French, but you should give up this belief were you to learn learning that Satie is compatriots with Verdi and Bizet (as in the original case) and were you to learn that Satie is *not* compatriots with Verdi and Bizet (as in the alternative case).

Note that CAUTIOUS causes trouble not just for the strong theories of belief revision like (Alchourrón, Gärdenfors, and Makinson 1985), but also for weaker theories, such as those in (Lin and Kelly 2012) and (Goodman and Salow forthcoming): although these theories predict *different* counterexamples to PRESERVATION and, in the case of Goodman and Salow (forthcoming), to ANTICIPATION as well, they agree with the predictions these principles make with respect to Composers specifically.[5]

Moreover, as Schultheis (2018) argues, CAUTIOUS constitutes a counterexample to any theory that accepts the following natural principle connecting credence to belief:

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[5]Why is this? According to both theories, one is justified in believing one is in the one of the most plausible worlds compatible with one's evidence, where this ordering of plausibility is determined by one's subjective probabilities. For instance, Lin and Kelly say that w is more plausible than w' just in case w is sufficiently more probable than w'. (Even though Goodman and Salow's (forthcoming) approach is notably weaker than Lin and Kelly's, both approaches work similarly enough for our purposes.) Therefore, to capture the claim that at t_1 you can justifiably believe Satie is French, the world in which all three composers are French must be sufficiently more probable than the world in which they are all Italian. The issue is that, at t_2, the ratio of probabilities between these two possibilities does not change, meaning that the all-French possibility will remain more plausible than the all-Italian possibility, meaning that at t_2 one is, contra CAUTIOUS, justified in believing all three composers are French.

LOCKEAN SUPERVENIENCE

If two agents A_1 and A_2 are justified in having the same credence with respect to p, then A_1 is justified in believing p iff A_2 is justified in believing p.[6]

For, plausibly, the following two facts should hold about your credences at t_1:

(i) At t_1, you should be 50/50 as to whether Verdi is Italian or French, as you should be as to whether Bizet is Italian or French.

(ii) At t_1, your confidence that Satie is French should be probabilistically independent from the claim that Verdi/Bizet is Italian/French.

Schultheis (2018, pp. 36-37) proves that if (i) and (ii) hold, then your credence that Satie is French cannot change on learning, at t_2, that all three composers are compatriots. As CAUTIOUS nevertheless says you should give up your belief that Satie is French at t_2, LOCKEAN SUPERVENINCE must be false.

CAUTIOUS may look innocent. Yet, if it's correct, epistemological anarchy ensues.

2 Bold Composers

Anarchy isn't for everyone. The anarchy wreaked by CAUTIOUS will therefore instead incline many towards skepticism about CAUTIOUS itself. You can, for instance, imagine the hard-nosed Lockean objecting: 'You haven't demonstrated a counterexample to LOCKEAN SUPERVENIENCE; rather, Schultheis's observations merely serve as a proof that CAUTIOUS is mistaken. Since the probability that Satie is French does not drop after learning all three composers are compatriots, you should continue to believe that Satie is French at t_2, and accordingly form the belief that all the composers are French."

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[6]There are various reasons why philosophers might deny LOCKEAN SUPERVENIENCE. (Buchak 2014) suggests agents with the same credences may differ in what they are justified in believing if those agents possess evidence of different kinds (e.g. testimonial evidence versus purely statistical evidence). (Fantl and McGrath 2009) suggest this can happen when the *practical stakes* for the relevant agents are different. However, for counterexamples like these there is a natural weaker principle: *holding fixed* the same kind of evidence, the practical stakes, etc., LOCKEAN SUPERVENIENCE holds. Composers promises to deliver a counterexample to even these restricted versions of LOCKEAN SUPERVENIENCE .

This reaction isn't obviously unreasonable.[7] If something like this hard-nosed Lockean reply is correct, then, contra CAUTIOUS, your beliefs ought to change as follows:

BOLD
In Composers, at t_1, you should suspend judgement on whether Verdi and Bizet are both Italian or both French, but you should continue to believe Satie is French (as with CAUTIOUS). However, at t_2, you should believe that all three composers are French.

And this pattern of belief revision is not obviously irrational. After all, it *is* more plausible that you were wrong about just one composer (Verdi) rather than two (Bizet and Satie), so perhaps the correct response really is to conclude you were just wrong about Verdi and hence that all three composers are French.

Still, this reply also feels a little "one philosopher's modus ponens is an other's modus tollens." If Composers does not provide a counterexample to LOCKEAN SUPERVENIENCE, what could?[8] The case for this reaction would be strengthened if there was more one could say in support of the claim that the intuitions supporting CAUTIOUS rest on a mistake.

There *is* more one can say. In making a point about an unrelated literature (they are responding to (White 2009)), Hart and Titlebaum (2015) consider:[9]

> **Urn Matching.** Urn X contains 50 black balls and 50 white balls. Urn Y contains 70 black balls and 30 white balls. I randomly select one ball from each urn. Then, without showing you the balls, I tell you that the two balls have the same color. You have all the relevant background information. What's the probability that the drawn balls were black?

Hart and Titlebaum write:

> Initially your credence that the X ball would be black was 0.5, and your credence that the Y ball would be black was 0.7. Since these credences must now come together somehow [since you know one

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[7]Goodman and Salow (forthcoming, §4) express a similar sentiment regarding the proposed counterexample to ANTICIPATION, which they discuss under the name "Π−'. Though note that their approach predicts other failures of ANTICIPATION (forthcoming, §8, §10).

[8]Though see footnote 6.

[9]See also (Titelbaum and Hart 2020).

ball is black iff the other is black], most people to whom we pose Urn Matching - even probabilistic sophisticates - answer somewhere between 0.5 and 0.7. (Hart and Titelbaum 2015, p. 254)

Yet this answer is incorrect. One's credence that the ball is black should *remain* 0.7. In general: "When a biconditional between two-independent propositions is learned and one of them had a prior probability of $\frac{1}{2}$, the posterior value equals the prior of the other proposition" (Hart and Titelbaum 2015, p. 255). Hart and Titlebaum suggest that most people are oblivious to this, and so are inclined to make probabilistic errors when updating on biconditionals.

Here's my point: at t_2 in Composers you are *also* asked to update on a biconditional—the proposition that all three composers are compatriots is equivalent, given your evidence, to the proposition that Verdi is French iff Satie is French. So, since we already have reason to believe people are bad in determining what doxastic attitudes one should have in cases involving updates on biconditionals, we have independent reason to think that people are going to make mistakes with respect to the right rational response in Composers. We therefore appear to have good and independently motivated grounds for dismissing Cautious.

This is an attractive view. It used to be *my* view. But I would now say it is only the second best view. My central reservation is that it fails to capture the following observation from Stalnaker:[10]

> A bold believer might reason as follows: the hypothesis of two independent errors is less plausible than the hypothesis of one error, so I should conclude that [all three composers are French]. A more cautious believer might grant that this possibility is more likely, but refrain from reaching a definite conclusion. This more cautious policy does not seem unreasonable. (Stalnaker 2019, p. 65)

That is, there is an intuitive sense in which the choice between Cautious and Bold is a *permissive* one. Neither is flatly irrational, but rather, which reaction one should have depends on how bold an epistemic agent we are considering. The bolder of us can reasonably conclude that we were wrong about only one composer, yet it is also not unreasonable for the more cautious of us to leave open that we were wrong about two.

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[10]Quote is edited to fit Composers. Stalnaker is here discussing a structurally analogous case.

The problem with the suggested approach is that it instead rejects Cautious as an outright irrational reaction. It's as if those in favour of Cautious just can't do the math. That really does not seem right. On the face of it, those who endorse Cautious are not committing anything resembling a *fallacy*; rather, they are just being more, well, cautious. It is therefore desirable to find an approach which can capture Stalnaker's observation that there appears to be a permissive element regarding the choice between Cautious and Bold.

In §4, I'll outline an approach that does exactly that. First, however, I should examine another influential approach to capturing the Cautious judgements.

3 Incomparable Composers

Lin (2019) outlines the following influential model of Composers.[11] Here's the basic idea. One is justified in believing p if p is true in all the most plausible worlds compatible with one's evidence. But it is not always possible to compare the plausibility of two worlds. If w is implausible in one way, yet w' is implausible in a different way, w and w' may be *incomparable*. Arguably, such a situation arises in Composers: a world in which you're wrong about just Verdi is not more plausible, but *incomparable*, to a world in which you're wrong about both Bizet and Satie. Both worlds are implausible, but implausible in *different ways*. After all, recall that these beliefs are stipulated to be formed on independent bases. On this approach, to say that it's more plausible that you were mistaken just about Verdi would be like saying that a world in which your partner lied to you is more plausible than a world in which you both mistakenly thought you could smell coffee and you misremembered the time of your job interview.

We can demonstrate what this approach says about Composers diagrammatically, with the following conventions: 'ABC' is the possibility in which Verdi, Bizet and Satie are, respectively, nationalities A, B and C, 'I' for Italian, 'F' for french; an arrow from one node to another says that the former is at least as plausible than the latter; arrows that can be inferred from reflexivity or transitivity are omitted. We stipulate that ABC is at least as plausible DEF iff any mistake you make in ABC you also make in DEF. We get the following diagram for your beliefs at t_0:

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[11](Boylan and Schultheis 2021) outline a similar model to account for Ginsberg's (1986) original case concerning counterfactuals.

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                    (IFF)
                  ↙   ↓   ↘
        (FFF)      (IIF)      (IFI)
          │    ✕        ✕       │
          ↓  ↙    ↘  ↙    ↘    ↓
        (FIF)      (FFI)      (III)
            ↘        ↓        ↙
                    (FII)
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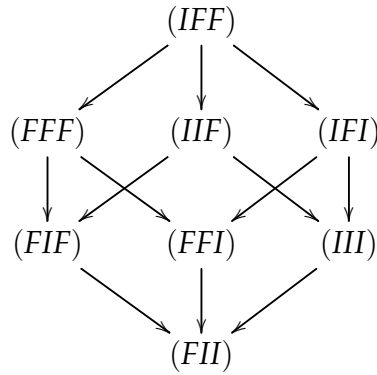
Figure 1: Incomparability model at t_0.

Since *IFF* is the most plausible world, you believe Verdi is Italian, and that Bizet and Satie are both French at t_0. On learning that Verdi and Bizet are compatriots at t_1, we get:
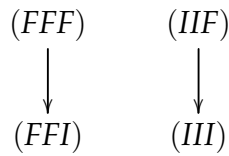
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        (FFF)        (IIF)
          │            │
          ↓            ↓
        (FFI)        (III)
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Figure 2: Incomparability model at t_1.

Since *FFF* and *IIF* are now the most plausible worlds (which are pairwise incomparable), you are ambivalent about Verdi and Bizet's nationality, but still believe Satie is French. Upon learning that all three composers are compatriots at t_2, you are left with two incomparably plausible worlds, *FFF* and *III*:

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        (FFF)        (III)
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Figure 3: Incomparability model at t_2.

Since neither world is less plausible than any other word compatible with your evidence, both are compatible with your justified beliefs. Per Cautious, you should be ambivalent as to whether the composers are all French or all Italian.

While elegant, my central complaint about this approach is the same as that discussed in §2: it *also* fails to capture Stalnaker's observation that the choice between Cautious and Bold is permissive. For once we introduce incomparabilites, Cautious comes out as the uniquely rational pattern of belief

revision. But, again, it does not seem like those who endorse BOLD commit any more of a fallacy than those who endorse CAUTIOUS do.

There is, however, a potential reply to my concern. Perhaps what marks the permissive difference between those who prefer CAUTIOUS those who prefer BOLD comes down to a further permissive difference as to whether they take worlds in which they have made different kinds of mistakes to be incomparable. If, for instance, you think worlds in which you made just one mistake is exactly as plausible as a world in which you also make just one mistake but about a different composer, we end up with the following, quite different model of Composers at t_0 and t_1:
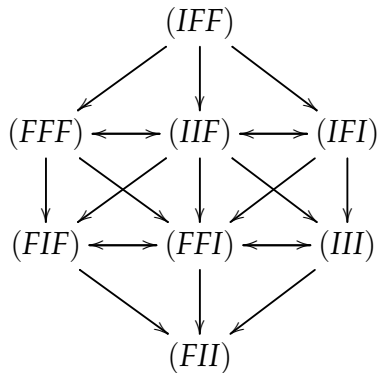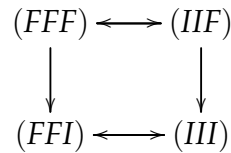

Figure 4: Comparability model at t_0.

Figure 5: Comparability model at t_1.

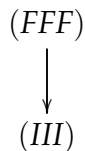Crucially, since *FFF* is now *more plausible* than *III*, we end up with the following model at t_2:

Figure 6: Incomparability model at t_1.

Hence, learning that all three composers are compatriots will put you in a position to justifiably believe you are in (*FFF*), per BOLD. So, can't this approach capture the permissive element of Composers after all?

The problem is that it can't naturally be extended to account for permissibility once we complicate the case. Consider:

Four Composers. You justifiably believe (at time t_0) that Verdi is Italian, Bizet is French, Satie is French, and Boulanger is French. These four beliefs are formed on equally strong, independent bases. At t_2, you learn that *all four* composers are compatriots.

The choice for you at t_2 is now between the possibilities that you were wrong only about Verdi, or that you were wrong about all three of Bizet, Satie, and Boulanger. If, like with the original composers, you take worlds in which you make different kinds of mistakes to nevertheless be comparable, this approach will predict that at t_2 you should believe Verdi is French: one mistake is more plausible than three. If, on the other hand, you take worlds in which you make different kinds of mistakes to be incomparable, then at t_2 you'll be ambivalent as to whether all four composers are French or all four are Italian: the plausibility of the situation in which you were wrong about Bizet, Satie and Boulanger is just incomparable to the plausibility of the situation in which you made the different kind of mistake of being wrong about Verdi.

But there is surely some middle ground here. It would be perfectly reasonable to, in the original Composers case, hesitate and leave open the possibility that you were mistaken about two composers, yet nevertheless in Four Composers conclude that you must have been mistaken about just Verdi rather than *three* composers. The incomparability approach cannot naturally account for this. Generally, no matter the number n of composers we introduce, an agent either treats cases with different kinds of mistakes as comparable, in which case they always conclude they were wrong about just Verdi, or the agent treats cases with different kinds of mistakes as incomparable, in which case they, no matter the value of n, leave it open that they may have been mistaken about n composers. Yet there must be some permissible middle ground between these two extreme reactions.

I'll now outline an approach that captures the permissive element of Composers without succumbing to this kind of objection.

4 Composers Composed

If we can't predict CAUTIOUS by appealing to incomparabilities, how else can we do it? Here's a clue. The focus of my argument for CAUTIOUS in §1 was not with the proposition *Satie is French*; I did not (falsely) claim that the probability of this proposition drops at t_2. Rather, the focus was on *the possibility* that

all three composers are Italian. I claimed that this is a possibility you are not entitled to rule out at t_2. Both Stalnaker's (1994, p. 19) and Schultheis's (2018, p. 32) arguments in favor of CAUTIOUS are similar in this respect. Notably, it's possible for the possibility that *all three composers are Italian* to increase, even if the probability of the proposition *Satie is French* does not drop.

A theory of justified belief that focuses on possibilities, rather than propositions, may therefore have a shot at predicting CAUTIOUS. A natural such theory is as follows. When is one entitled to rule a possibility out? When that possibility is sufficiently unlikely. Following this idea, we can define justified belief as follows: the strongest proposition I can justifiably believe is that I am in one of the possibilities I am not in a position to rule out.[12] Indeed, this answer fits nicely with the popular view that the function of belief is to simplify reasoning: beliefs allow us to ignore those possibilities that are so unlikely we need not bother thinking about them.[13]

We can use these ideas to model Composers and predict CAUTIOUS. The key observation is that, although at t_2 the probability that Satie is French does not decrease, the probability in the possibility that they are all Italian increases. Let W be a set of possibilities, consisting of triplets ABC, with each place indicating the nationality—either Italian ('I') or French ('F')—of Verdi, Bizet and Satie respectively:

- $W = \{IFF, FFF, IFI, IIF, FIF, FFI, III, FII\}$

Let P be the following probability distribution over W, representing your credences at t_0:

- $P(IFF) = 0.729$

- $P(FFF) = P(IFI) = P(IIF) = 0.081$

- $P(FIF) = P(FFI) = P(III) = 0.009$

- $P(FII) = 0.001$

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[12]Levi (1967) defends a theory of belief that entails the same predictions with respect to what one is justified in believing at a time. However, Levi's approach is importantly different to the one defended here with respect to how one's justified beliefs change on receiving new evidence, which prevents it from predicting CAUTIOUS. See fn.14 for details.

[13]See (Harsanyi 1985), (Lance 1995), (Lin 2013) and (Ross and Schroeder 2014); see (Staffel 2019) for recent criticism.

Figure 7: New model at t_0.

These values were chosen arbitrarily within two constraints: (i) worlds in which you have more false beliefs at t_0 are less probable than worlds in which you have less false beliefs at t_0; and (ii) propositions specifying the nationality of a composer are probabilistically independent from propositions specifying the nationality of a different composer.

Where $e_{t_n} \subseteq W$ is your evidence at t_n, define your strongest justified belief with evidence e_{t_n}, B_{t_n}, as follows. Where $P(p \mid e_{t_n}) = \frac{P(p \& e_{t_n})}{P(e_{t_n})}$, and $0 \leqslant \tau \leqslant 1$:

- $B_{t_n} = \{w \in W : P(w \mid e_{t_n}) \geqslant \tau\}$

In words: your strongest justified belief at t_n, B_{t_n}, is that you are in one of the possibilities with sufficiently high probability. You are justified in believing p at t_n iff B_{t_n} entails p.

Setting $\tau = 0.1$, we get the following predictions for Composers:

- At t_0, you believe you are in the only possibility with sufficiently high probability, *IFF*. (See Figure 7 for an illustration.)

- At t_1 you learn that Verdi and Bizet are compatriots, so $e_{t_1} = \{FFF, IIF, FFI, III\}$. The updated probabilities of these four remaining possibilities are:
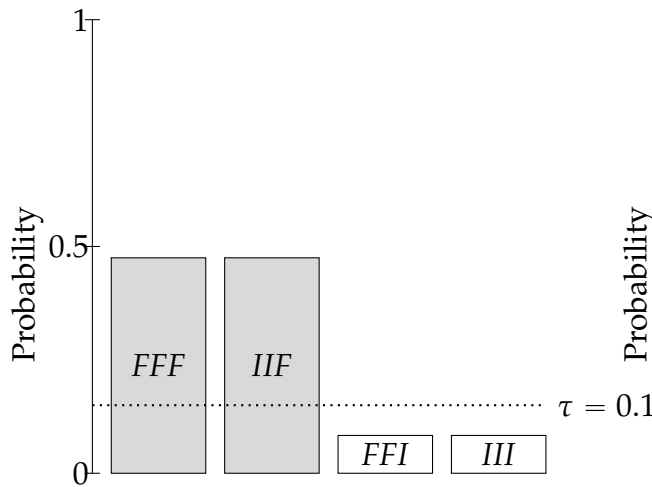
    - $P(FFF \mid e_{t_1}) = P(IIF \mid e_{t_1}) = 0.45$;
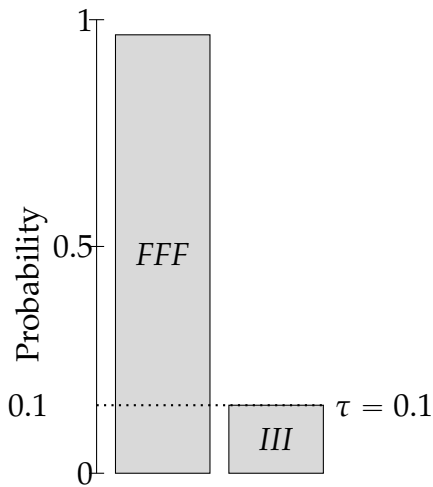
13

Figure 8: New model at t_1.

Figure 9: New model at t_2.

– $P(FFI \mid e_{t_1}) = P(III \mid e_{t_1}) = 0.05$;

So at t_1 your strongest belief is *FFF* ∨ *IIF*—that Satie is French yet Verdi and Bizet could either be both French or both Italian—in alignment with Cautious (and Bold). (See Figure 8.)

- At t_2, you learn that all composers are compatriots, so $e_{t_2} = \{FFF, III\}$ and our updated probabilities are:

  – $P(FFF \mid e_{t_2}) = 0.9$

  – $P(III \mid e_{t_2}) = 0.1$

Since *III* now has probability of at least τ, it is a member of B_{t_2}, meaning your strongest belief is *FFF* ∨ *III*, per Cautious but contra Bold. (See Figure 9.)

We therefore have an independently plausible theory of justified belief that naturally predicts Cautious.[14]

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[14]Why can't Levi (1967), mention in fn.12, make these predictions? Here's a quick overview of Levi's view. Let q be a constant measuring an agent's 'epistemic boldness'—their inclination to form informative beliefs at the cost of forming false beliefs—and let $cont_e$ be a function taking propositions to a measure of their content, that depends on an agent's evidence e. Levi's theory states that $w \in W$ is a member of an agent's belief set iff:

But isn't the choice between Cautious and Bold meant to be permissive? Well remembered. The present approach can account for this by supposing that, for agents who are more epistemically "bold"—agents who are inclined to rule out more probable possibilities than more cautious agents are—τ ought to be set higher. Setting $\tau = 0.2$, while your justified beliefs at t_0 and t_1 are no different from those above, at t_2 *III remains* a sufficiently unlikely possibility, meaning you can justifiably believe *FFF*, as per Bold.

That makes the current approach at least as good as the incomparability approach discussed in §4. So, how does it fair with respect to Four Composers? Better. Setting our value of τ back to 0.1, you ought to in Composers leave open at t_2 the possibility that all three composers are Italian. Yet, on considering a natural extension of the probability function P as applied to Four Composers, it is predicted that you ought to rule out the possibility that all four composers are Italian. For the probability at t_2 of the possibility, in Four Composers, that all four composers are Italian will be much lower than the probability at t_2, in Composers, that all three composers are Italian. The following footnote contains the details.[15]

(*) $P(w) \geqslant q \times cont_e(W \backslash \{w\})$.

Levi defines the content of a proposition to be the proportion of worlds in $W \backslash e$ inconsistent with it. At t_1, $W \backslash e_{t_1}$ has four elements, meaning that by (*), at t_1 one leaves open those worlds w which are such that $P(w) \geqslant q \times \frac{1}{4}$. Hence, if we set $0.2 < q \leqslant 1.8$, at t_1 you'll leave open just *FFF* and *IIF*, per Cautious. However, since $W \backslash e_{t_2}$ only has two elements, by (*), at t_2 one leaves open those worlds w which are such that $P(w) \geqslant q \times \frac{1}{2}$. In order to be ambivalent about *FFF* and *III* at t_2, we therefore need q to be at most 0.2. There is therefore no single value of q that can be used, on Levi's approach, to vindicate Cautious. One way to respond is to suggest that Levi's measure of content should not be relativised to an agent's evidence e. However, this would have the strange upshot that a proposition such as $\{w_1\}$ would be just as informative for an agent with no evidence as it is for an agent whose evidence is $\{w_1, w_2\}$.

[15]A natural way to model **Four Composers**, following the lead of our model of Composers, is as follows. We first extend our set of worlds to include possibilities in which Boulanger is French and possibilities in which she is Italian:

$$W^* = \{IFFF, FFFF, IIFF, IFIF, IFFI, FIFF, FFIF, FFFI, IIIF, IIFI, IIII, FIIF, FIFI, FFII, IIII, IFFF\}$$

We then naturally extend the probability distribution P to distribute over W^*, while still conforming to the constraints (i) and (ii) on p. 12 as follows. Where m_n, with $0 \leqslant n \leqslant 4$, is some world where you initially made n mistakes in **Four Composers**, let $P(m_0) = 0.6561$, $P(m_1) = 0.0729$, $P(m_2) = 0.0081$, $P(m_3) = 0.0009$ and $P(m_4) = 0.0001$. Where e is the proposition that all four composers are compatriots, only *FFFF* and *IIII* are not ruled out, and we have $P(FFFF \mid e) = 0.987$ and $P(IIII \mid e) = 0.01$. Hence, if we have $\tau = 0.1$, one is justified at t_2 in believing they only made one mistake, even though an agent with the same value for τ in

My approach also possesses additional virtues. For instance, it reveals additional judgements concerning Composers which are, on reflection, also perfectly reasonable. Setting $\tau = 0.05$, at t_0, you suspect you may be wrong about one of the composers' nationality; after learning at t_1 that you are wrong about at least one of them, you leave it open that you may be wrong about *two*. That is, you are always such that, if you know you have at least n mistaken sources, you leave it open that you have $n + 1$ mistaken sources. This is quite a cautious way to leads one's life, but it again does not seem unreasonable.

A second concerns Lockean Supervenience. As mentioned in §1, since this approach predicts Cautious, it predicts failures of Lockean Supervenience. However, note first that it does so in a way that can seem natural. For even if new evidence does not decrease the probability of a proposition p, it may nevertheless increase the probability of a currently ignored possibility w that conflicts with p. If the probability of w is increased to the extent that it can no longer be ignored, I must now pay attention to some $\neg p$ possibilities, and so I am no longer justified in believing p. Note second that the approach nevertheless preserves a weakened kind of supervenience: two agents with the same credences, and the same threshold τ, will not differ in what propositions they are justified in believing. One way to put this point is that, although belief does not supervene on credence *locally*—we cannot determine whether one can justifaibly believe p by simply looking at one's credence that p—it nevertheless supervenes on credence *globally*—with an appropriate threshold, we can determine whether one believes that p by looking at one's entire credence function.

A third virtue is that this account has further independent motivation. Pearson (ms) outlines a similar account of justified belief and argues that it provides the best account of the notorious coin flipping cases discusses in (Dorr, Goodman, and Hawthorne 2014) and (Goodman and Salow 2023).

On the other hand, the account is currently very simple, and as it stands faces two objections, which I'll now respond to. The first objection is that, on the suggested approach, what one is justified in believing looks objectionably sensitive to the fineness-of-grain of the worlds in W. Suppose, for instance, you learn that a coin will be flipped, and therefore begin to distinguish between worlds in W according to how the coin lands: each possibility in W will be split into two (one in which the coin lands heads, the other in which it lands tails) with each assigned half the probability of the original possibility from W.

Composers ought to, at t_2, leave it open that they made two mistakes.

Assuming the threshold t remains fixed, this means that more possibilities can be ruled out, meaning you may now form a stronger justified belief than before, say a belief that p, even if the fact that a coin has been flipped is irrelevant as to whether p.[16]

The most promising reply to this worry is to endorse a version of the view in which belief is *question-sensitive*.[17] So long as the flip of a fair coin *is* in fact irrelevant to the question-under-discussion, worlds distinguished by how the coin lands will belong to the same answer to the relevant question, and no objectionable belief change will occur on learning that a fair coin will be flipped. Formally, where Q is the salient question—a partition of W—and $[\cdot]_Q$ sends $w \in W$ to the cell of Q that w is a member of, we redefine one's strongest belief as sensitive both to one's evidence and to the relevant question as follows:

- $B_{t_n,Q} = \{w \in W : P([w]_Q \mid e_{t_n}) \geqslant \tau\}$

I deny that this solution to the problem is ad hoc. Indeed, it fits in nicely with the initial motivation given for the theory: that beliefs simplify reasoning. For partitioning a complex and large set of possibilities into fewer chunks is another natural way for agents to simplify their reasoning.[18]

The second objection is that, as it stands, the theory permits agents having justified beliefs in highly unlikely propositions. Suppose the threshold τ is $\frac{1}{3}$, and the relevant question Q consists of four answers, a, b, c and d, such that $Pr(a \mid e_{t_n}) = \frac{1}{3}$, yet $Pr(b \mid e_{t_n}) = Pr(c \mid e_{t_n}) = Pr(d \mid e_{t_n}) = \frac{2}{9}$. Here, $B_{e_{t_n},Q} = a$ and so the relevant agent is justified in believing a proposition they take to only be $\frac{1}{3}$ likely. A particularly acute version of this problem occurs if *none* of the answers to Q have probability of at least τ. In that scenario, $B_{e_{t_n},Q}$ is the empty set, and so it is predicted that our agent is justified in believing the contradiction.[19]

I'll fix this in a manner that broadly follows Pearson (ms), who in turn takes an approach similar to, but slightly different from, Hong (2023). We first

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[16]The problem is similar to one in Staffel's (2016) critique of (Leitgeb 2014).

[17]This is a common move nowadays. See (Leitgeb 2017), (Yalcin 2018), (Blumberg and Lederman 2020), (Hoek 2022), (Dorst and Mandelkern 2022), (Holguín 2022), (Hong 2023), (Goodman and Salow forthcoming) and (Pearson ms).

[18]The idea that coarse-gaining possibilities is a way for agents to simplify decision problems is also common in economics; see, for example: (Ahn and Ergin 2010), (Epstein, Marinacci, and Seo 2007) and (Gul, Pesendorfer, and Strzalecki 2017). See also (Dorst and Mandelkern 2022, pp. 611–12).

[19]While theories of weak belief, such as those in (Dorst and Mandelkern 2022) and (Holguín 2022), will be happy with justified beliefs in unlikely propositions, they will not be happy with justified beliefs in contradictory propositions.

specify a different threshold T which determines the minimum probability any proposition must have in order to be justifiably believed. However, in cases where τ is set high enough, the set of all possibilities that are at least τ-likely may not be at least T-likely. If so, one must retreat to some smaller threshold, $\tau^- < \tau$, such that the set of all τ^--likely possibilities *is* at least T-likely. This new set is then identified as the strongest proposition one is justified in believing. This strikes me as a natural solution to the problem. If you are usually comfortable ignoring worlds that are less than τ-likely, then on encountering a situation in which the set of τ-likely is itself not so probable, you'll more likely lower your standards and instead only rule out worlds that are less than τ^- likely, rather than, as the first-pass theory suggested, form a belief in a proposition that is highly unlikely.

In sum, we have a theory of justified belief that is well-motivated, natural, and stands up to scrutiny. Moreover, it provides a model of Composers that delivers various attractive results. Composers has been composed.

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